Shot noise in x-ray measurements with *p-i-n* diodes

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The importance of shot noise is considered for situations in which *p-i-n* diodes monitor x-ray radiation. An expression for shot noise is derived in terms of the photon energy, the pair creation energy of the diode material, and the photocurrent. Statistical analysis shows that the Fano factor can be neglected for noise calculations. A lock-in amplifier measured the low frequency photocurrent noise from an unbiased silicon *p-i-n* photodiode that monitored radiation in the range of 6–16 keV at a synchrotron beamline. With ordinary electronic amplification and shielding, shot noise dominated other noise sources for photocurrents exceeding about 5 pA. [DOI: 10.1063/1.1947776]

PIN diodes have been established[1,2] as convenient and inexpensive detectors for soft x rays in the region below 20 keV. They differ from conventional photodiodes in that a PIN diode is made with the *p* and *n* layers separated by an "intrinsic" region, which is effective for converting an incident x-ray photon into a large number of electron hole pairs. Because of their high sensitivity and large dynamic range, PIN diodes are used for a variety of diagnostic applications at synchrotron facilities. One such use is in photodiode based beam position monitors,[3] which detect drift and low frequency pointing noise in a focused beam of x rays. As with all such high resolution observations, it is necessary to evaluate the measured noise in order to maximize signal to noise ratio. For position sensing of laser beams in the visible wavelength region, it has been shown[4] in an optimally designed measurement that the shot noise of photoelectrons can be the predominant source of noise which limits resolution. In this work, the importance of shot noise in PIN diodes for measuring x rays is considered, and the expectations are confirmed in a series of simple tests.

Electrical current flowing across a potential barrier, as in a diode or a vacuum tube, obeys the rules of quantum statistics. Because of this, the number of electrons that pass through the barrier during a given measurement interval will be subject to random statistical variations. In this case, the well known Schottky formula[5,6] for shot noise is applicable:

$$I_{\text{sh}} = \sqrt{2qIB}. \tag{1}$$

Here, I_{sh} is the shot noise expressed in units of amperes, q is the charge (C) carried by each shot event, I is the observed current (A), and B is the bandwidth (Hz) of the measuring system. In most instruments, q is equal to $e=1.6\times10^{-19}$ C, the fundamental electronic charge. For the mathematical derivation of this formula, current is modeled as a sequence of randomly occurring instantaneous pulses, so the measured current is equal to q multiplied by the rate of these pulses. Shot noise is naturally unavoidable and is "white," i.e., it has a power density that is constant as a function of frequency.

When a photodiode measures radiation with photon energy $(h\nu)$ much greater than the band gap energy of the diode material, it is possible for each photon to produce many electron-hole pairs. The exact number of charge carriers produced per photon is not deterministic, as there will be some statistical variance around an average value. Therefore, it may be necessary to modify the Schottky formula of Eq. (1). A technique to account for the variance in the number of charge carriers produced per photon is to interpret the accumulation of all the variably charged pulses as the superposition of many pulse sequences, with each sequence containing pulses of equal charge. Each such sequence can be considered to be an individual noise source, for which the Schottky formula holds. Because all of the pulses occur randomly and independently, these pulse sequences are uncorrelated. For an ensemble of uncorrelated noise sources, the square of total noise is equal to the sum of the squares of all individual sources.[5] Following this reasoning, one can square Eq. (1) and sum over n, the number of electron hole pairs created by each photon:

$$I_{\text{sh}}^2 = \sum_{n=1}^{\infty} 2q_n I_n B. \tag{2}$$

Under this notation, $q_n = n\,e$ is the charge associated with the creation of n electron-hole pairs, and I_n is the portion of the current that is associated with all pulses containing a charge equal to q_n. Writing $I_n = n\,e\,\dot{N}$, where $\dot{N}$ is the rate (s^{-1}) at which pulses carrying a charge of q_n are produced, gives:

$$I_{\text{sh}}^2 = 2Be^2 \sum_{n=1}^{\infty} n^2 \dot{N}. \tag{3}$$

In these terms, the total observed current is:

$$I = \sum_{n=1}^{\infty} I_n = e \sum_{n=1}^{\infty} n\dot{N}. \tag{4}$$

Comparing Eqs. (3) and (4) yields:

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$$\frac{I_{\mathrm{sh}}^2}{I} = \frac{2Be\langle n^2\rangle}{\langle n\rangle} \tag{5}$$

where $\langle n^2\rangle$ and $\langle n\rangle$ are, respectively, the mean values for n^2 and n. The equation can be written also as:

$$I_{\mathrm{sh}} = \sqrt{\frac{2BeI\langle n^2\rangle}{\langle n\rangle}}. \tag{6}$$

This result is analogous to the case of avalanche photodiodes, in which an applied bias potential converts a detected photon into a cascade of electrons, thus providing current gain. For avalanche photodiodes, the statistical variance of the number of electrons created per photon is incorporated into an "excess noise factor," which is defined[7,8] as the ratio $\langle M^2\rangle/\langle M\rangle^2$, where M is the photodiode gain. The particular design and construction of an avalanche photodiode determines the value of this excess noise factor.

If high energy x rays are incident upon a semiconductor that is optically thick, the pair creation energy ε determines the value of $\langle n\rangle$, the average number of electron hole pairs created per photon. ε is a material parameter defined by $\langle n\rangle = h\nu/\varepsilon$ where $h\nu$ is equal to the photon energy of the incident x rays. For monochromatic radiation, the variance in n is governed by another material parameter, F, the Fano factor. The Fano factor is a constant with a value between 0 and 1, and is used primarily for determining the energy resolution of radiation detectors. It is defined[9] as the variance in the number of pairs divided by the average number of pairs:

$$F = \frac{\langle n^2\rangle - \langle n\rangle^2}{\langle n\rangle}. \tag{7}$$

Using the definitions of ε and F allows Eq. (6) to be rewritten as an explicit expression for shot noise in terms of material parameters and directly measured quantities:

$$I_{\mathrm{sh}} = \sqrt{2BeI\left(F + \frac{h\nu}{\varepsilon}\right)}. \tag{8}$$

The first term within the parentheses of the above equation, F, is less than unity, while the second term generally is much greater than unity. For example, consider a situation in which a silicon detector monitors x rays with photon energy of $h\nu = 10$ keV. Silicon has measured[9] values of $\varepsilon = 3.63$ eV, and $F = 0.115$ at room temperature. Inserting these numbers into the above equation shows that the Fano factor alters the calculated value for shot noise by only 0.002%, an amount that is beyond the precision needed for noise evaluation. Therefore, it is reasonable to omit the Fano factor from Eq. (8) and rewrite the expression for shot noise as:

$$I_{\mathrm{sh}} = \sqrt{2BeI\left(\frac{h\nu}{\varepsilon}\right)}. \tag{9}$$

This is a simple and intuitive result, equivalent to Eq. (1) with the charge per shot being a deterministic value of $q = e(h\nu/\varepsilon)$. Thus, when photodiodes are used for monitoring x rays, the increased current provided by the high energy of the photons can be considered as "noiseless gain." This characteristic makes it different from the current gain provided by avalanche photodiodes, which requires accounting for an excess noise factor. Equation (9) can be useful for predicting shot noise in measurement systems where the photocurrent caused by monochromatic radiation is being recorded in real time. For x rays that are not monochromatic, Eq. (6) gives a more general expression for shot noise. The shot noise limit is a statistical limit, and the highest signal to noise ratio attainable for such a current measurement system is equal to that attainable from an ideal electronic photon counting system, which records and counts each individual photoelectric pulse produced in the detector.

In order to verify the shot noise formula of Eq. (9), I performed a series of three tests, exposing a *p-i-n* diode to a constant source of x-ray radiation. These tests occurred at beamline number 8.2.1 of The Advanced Light Source, a synchrotron facility. This beamline[10] provides a source of monochromatic x rays in the range of 5–18 keV, with a stabilized[11] intensity. The tests were made with a prototype photodiode based beam position monitor, in which a United Detector Technology (UDT) model S-100VL silicon photodiode was exposed to x rays scattered from a metal film (0.5 μm of Cr deposited onto a 25 μm thick sheet of Kapton polyimide). This *p-i-n* diode is typical of those used for detecting x rays. It has a surface area of 94 mm^2, an impedance of 1 MΩ, and a retail cost of less than \$20. The photocurrent was converted into a voltage signal by a Stanford Research Systems model SR570 low-noise current preamplifier, operating with zero bias and a gain setting of 10 nA/V. For noise measurements, I used a lock-in amplifier (Stanford Research Systems model SR830) operating with an equivalent noise bandwidth of 4.2 Hz. This instrument has a digital signal processor which computes noise via the mean absolute deviation method.

In one test, I varied the measurement frequency while keeping the photon energy fixed at $h\nu = 12.1$ keV. Scattered radiation incident on the diode produced a photocurrent of $I = 8.7$ nA. For this current and photon energy, Eq. (9) indicates that $I_{\mathrm{sh}}/\sqrt{B} = 3.0$ pA/$\sqrt{\mathrm{Hz}}$. The measured noise for nine frequencies, ranging from 1 to 49 Hz, matched the calculated value for shot noise, to within the estimated measurement accuracy of ±10%. In another test, I varied the photon energy from 6 to 16 keV, in increments of 2 keV, with the measurement frequency kept at 10 Hz. The photocurrent fluctuated from 0.7 to 8.3 nA. As with the first test, the observed noise was consistent with the shot noise formula of Eq. (9). Thus, it was clear that shot noise was the primary source of noise in the measurement system.

For a third test, I inserted various combinations of metal foil filters into the beam path, thereby reducing the photocurrent down to a minimum of $I = 0.19$ nA. Photon energy was set to 12.1 keV, and noise was measured at a frequency of 10 Hz. As with the other tests, shot noise accounted for the total observed noise, to within measurement accuracy. When the beam was completely blocked ($I = 0$), the observed noise was equal to 70 fA/$\sqrt{\mathrm{Hz}}$, which represents the internal noise of the measurement system. For the particular photon energy and detector material of this work, this is equivalent to the shot noise from a photocurrent of 5 pA. This quantity is useful to know when preparing for a real measurement. If the photocurrent being measured is much greater than this value, efforts to improve the shielding of the detector or cables will not result in significant increase in the signal to noise ratio, nor will an improvement of the electronic amplification system. For measurement of currents comparable to or smaller than this value, such efforts may provide benefits.

The internal noise from a photodiode amplifier system depends[12,13] on both the electronics of the amplifier and the electrical properties of the photodiode. The current amplifier used in this work is a commonly available commercial product, so the measured noise of level of 70 $\mathrm{fA}/\sqrt{\mathrm{Hz}}$ can be considered typical of a value that is readily obtainable. The essence of such a current amplifier is an operational amplifier with a feedback resistor. An important noise source is the Johnson noise of the feedback resistor. The Johnson noise, also called thermal noise, in units of $(\mathrm{V}/\sqrt{\mathrm{Hz}})$, is $V_{\mathrm{noise}}/\sqrt{B} = \sqrt{4kTR}$, where $k=1.38\times 10^{-23}$ J/K is Boltzmann's constant, T is the temperature (K), and R is the resistance (Ω) of the feedback resistor. Another important noise contribution in the system is from the input noise voltage of the operational amplifier within the current amp, in this case an Analog Devices model AD546. Because the noise voltage is applied across the finite impedance of the photodiode, some current noise will be passed through the input and will be susceptible to the gain of the feedback resistor. The noise contribution from this effect can be reduced by using a photodiode with greater impedance.

In some measurements, the frequencies of interest for signal and noise will be considerably lower than those measured in this work. For example, steady state photocurrent measurements can occur over a long duration, in which case $1/f$ noise becomes an important practical consideration. Theoretically, the various mechanisms[14,15] of $1/f$ noise are beyond the scope of this work. Although $1/f$ noise will be minimized in a well designed instrument, it cannot be eliminated completely, and its existence implies that noise becomes infinite as f approaches zero. However, in a real experiment, the amount of time available for measurement is limited. Therefore, the effect of $1/f$ noise also is limited.

Johnson noise, $1/f$ noise, and other sources of electrical noise, are characteristics of a measurement system, but shot noise is an unavoidable effect. The objective of instrumental design should be to reduce the extraneous sources of noise until shot noise is the predominant source. In many situations, this will be possible.

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